Towards quantum metrology with NOON states enabled by ensemble-cavity interaction

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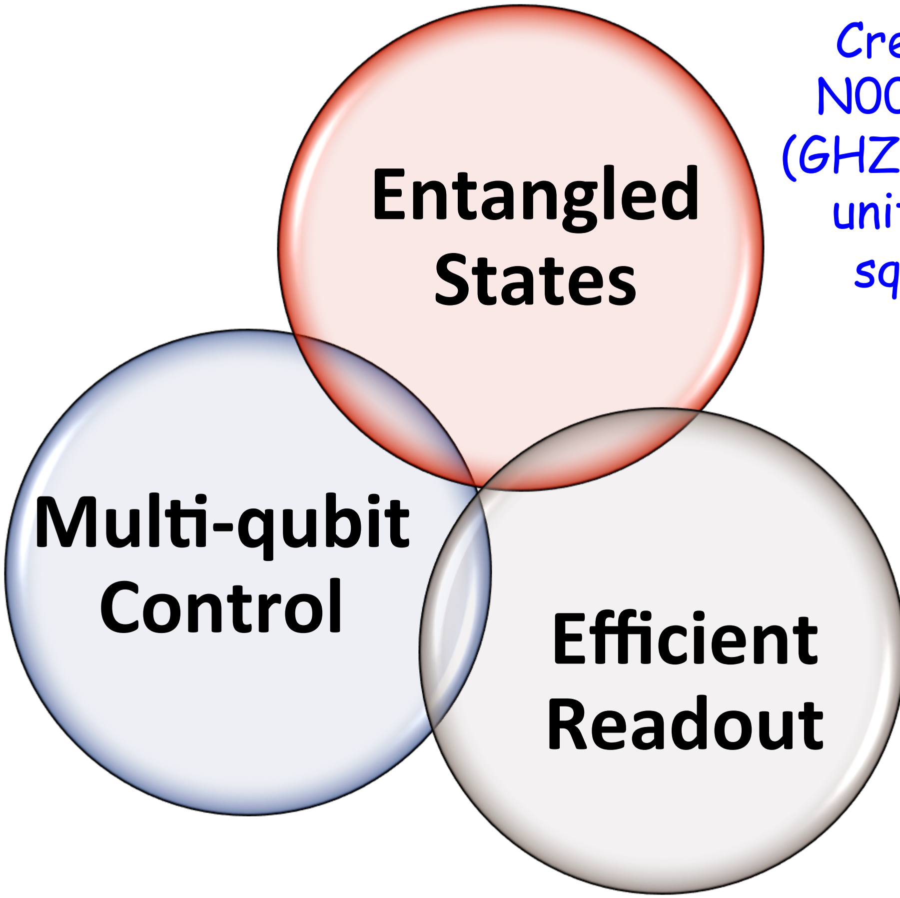
Outline

- NOON state (GHZ state) metrology at the Heisenberg limit;
- Creation of spin NOON states in mesoscopic atomic ensembles by spin squeezing around the Bloch sphere;
- Cavity spin squeezing in atomic ensembles: effective infinite-distance atomic spin-spin interaction mediated by light;
- Quantum eraser for atom-light entanglement: towards unitary cavity spin squeezing;
- Counting atoms in mesoscopic atomic ensembles: Experimental demonstration of readout capability for Heisenberg-limited sensing.

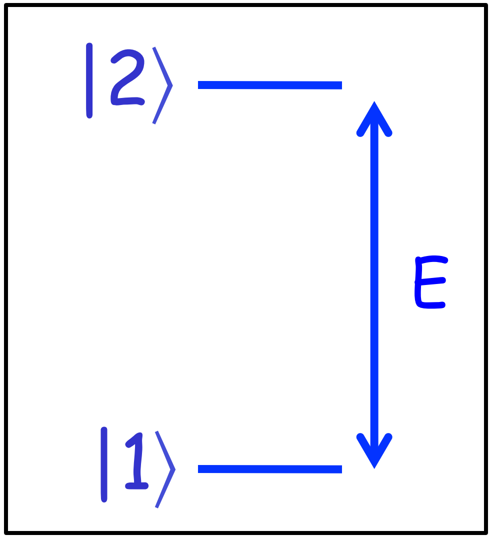
Program Themes



Creation of NOON states (GHZ states) by unitary spin squeezing

Measurement of mesoscopic atomic ensembles with single-atom resolution

Single-atom clock: Ramsey sequence



- Initialize atom in state $|\Psi_i\rangle = |1\rangle$
- Prepare atom in superposition state by π/2 pulse.

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$$

- Let state evolve for time T

$$|\Psi(T)\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + \exp\left(-iET/\hbar\right)|2\rangle\right)$$

- Apply second π/2 pulse to transform phase $\phi = ET/\hbar$ into amplitude

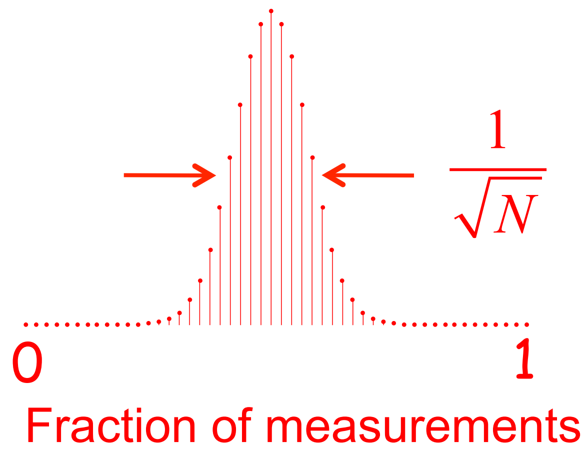
$$|\Psi_f\rangle = \cos\phi|1\rangle + \sin\phi|2\rangle$$

- Measure atomic state |1⟩ or |2⟩.
- Determine ϕ and hence $\nu=\phi/T$ from probabilities $\sin^2\phi$, $\cos^2\phi$ to observe atom in states |1⟩, |2⟩.
- At point of optimum sensitivity $\phi=\pi/4$ the measurement corresponds to tossing a coin with possible outcomes |1⟩, |2⟩: **quantum noise**.

Measurement precision

Repeated measurement with N independent atoms:

Binomial distribution ≈ Gaussian distribution

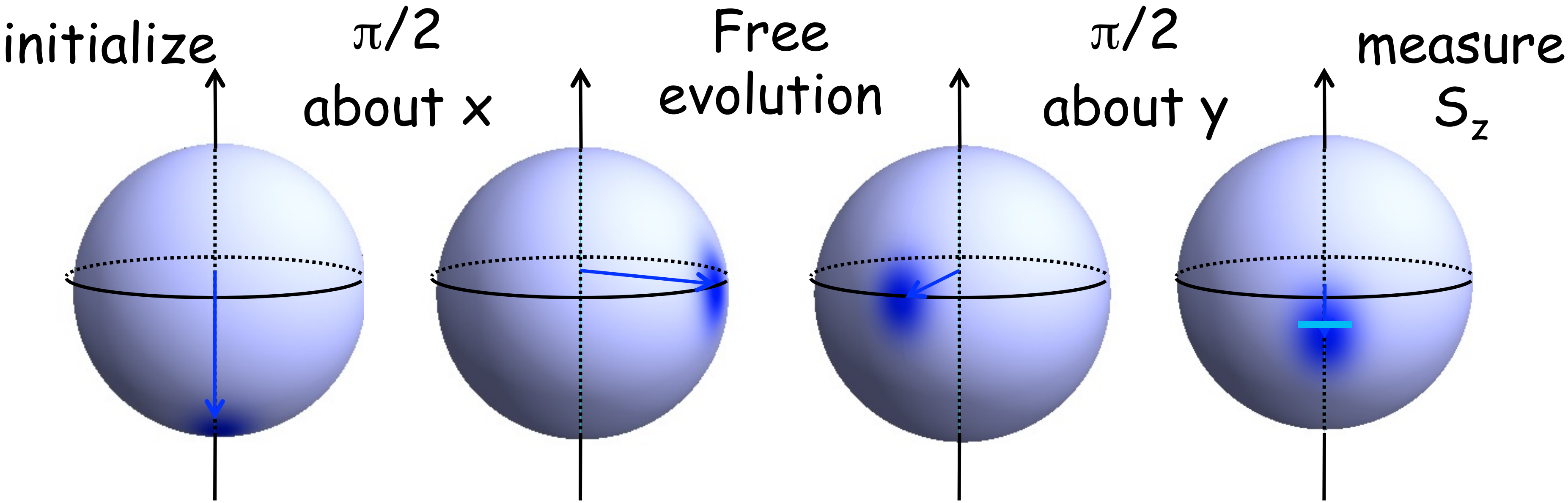


Signal $\propto N$ quantum projection noise $\propto \sqrt{N}$

Measurement precision scales as $1/\sqrt{N}$

Standard Quantum Limit

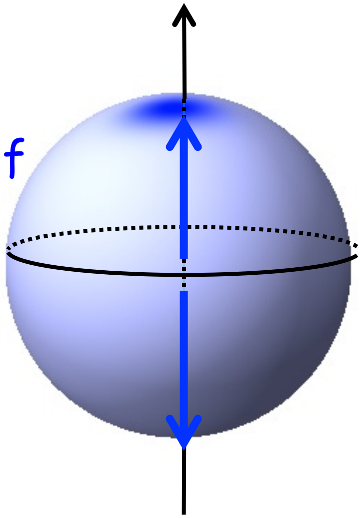
Clock operation in Bloch representation



Fuzzy area represents quasiprobability distribution for quantum projection noise (tossing N atomic coins) or angular-momentum uncertainty relations.

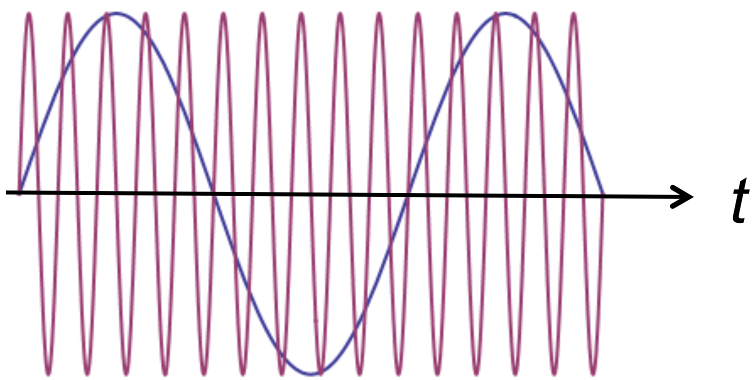
Interferometry with NOON states

NOON: (GHZ) state:
Superposition of two opposite coherent spin states



Let state evolve for time T

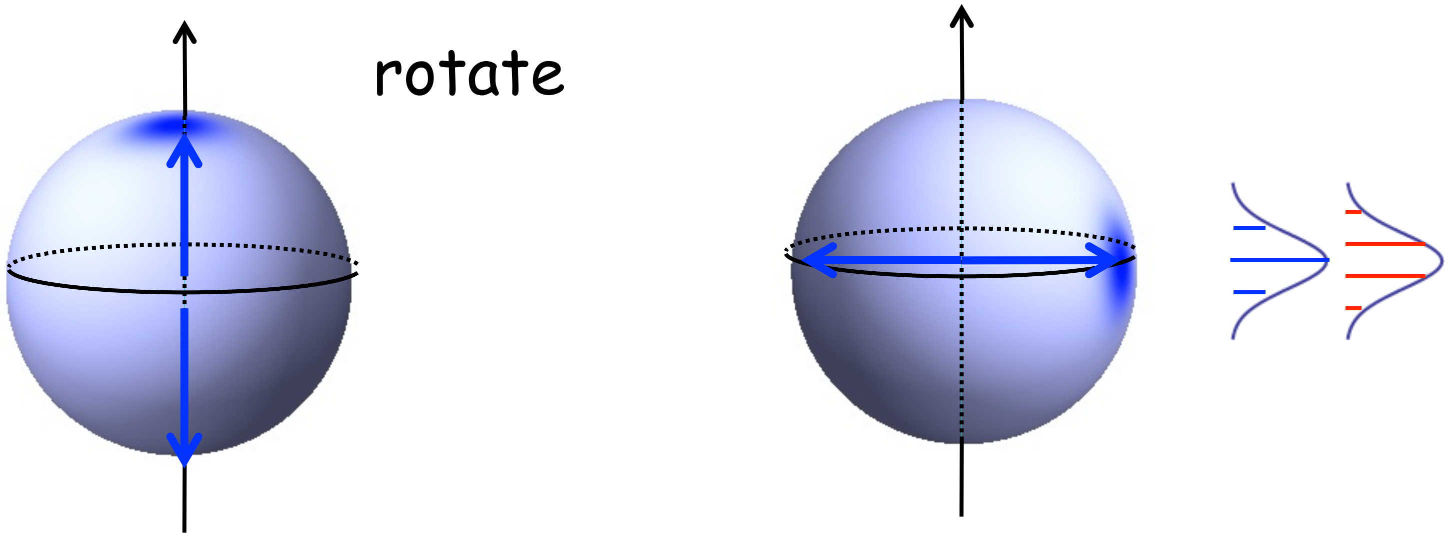
$$|\Psi(T)\rangle = \frac{1}{\sqrt{2}}\left(|11...1\rangle + \exp\left(-iNET/\hbar\right)|22...2\rangle\right)$$



Phase evolution *N* times faster for NOON state;

- How to generate NOON state?
- How to measure phase?

How to measure phase of NOON state



$|11\ldots1\rangle + \exp(-i\phi)|22\ldots2\rangle$

$|x\rangle + \exp(-i\phi)|-x\rangle$

$$\sum e^{-m^2/2S}|m\rangle + \exp(-i\phi)\sum(-1)^m e^{-m^2/2S}|m\rangle$$

For $e^{i\phi} = \pm 1$ only even/odd parity S_z states populated: phase of NOON state maps onto parity of S_z

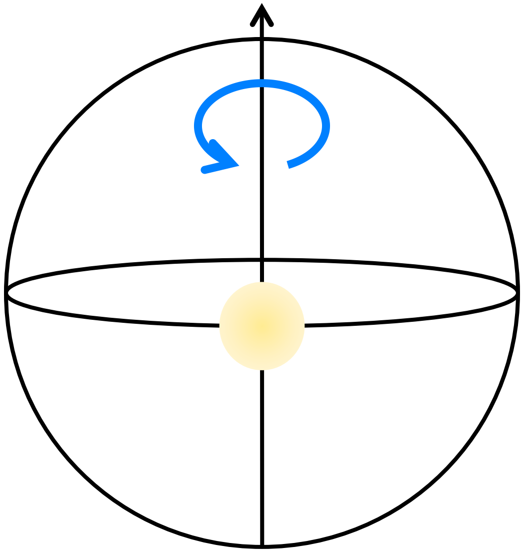
Cavity squeezing:

effective spin-spin interaction between distant atoms mediated by light

Spin squeezing by one-axis twisting

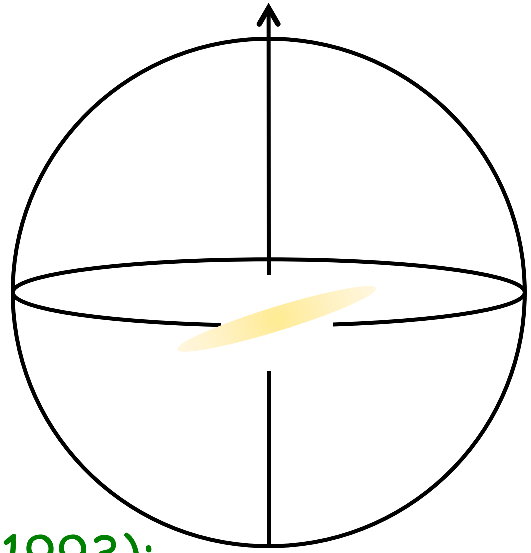
$$H = S_z$$

generates rotations about the z axis by angle θ $e^{i\theta S_z}$



$$H = S_z^2$$

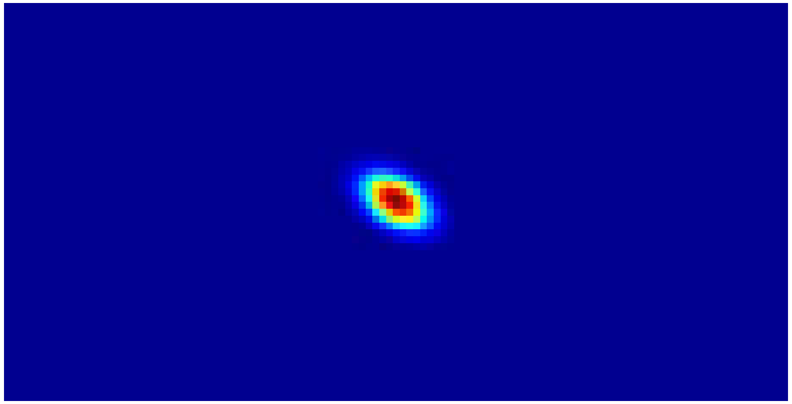
generates rotations about the z axis by S_z-dependent angle $\theta(S_z)$ $e^{i\theta(S_z)S_z}$



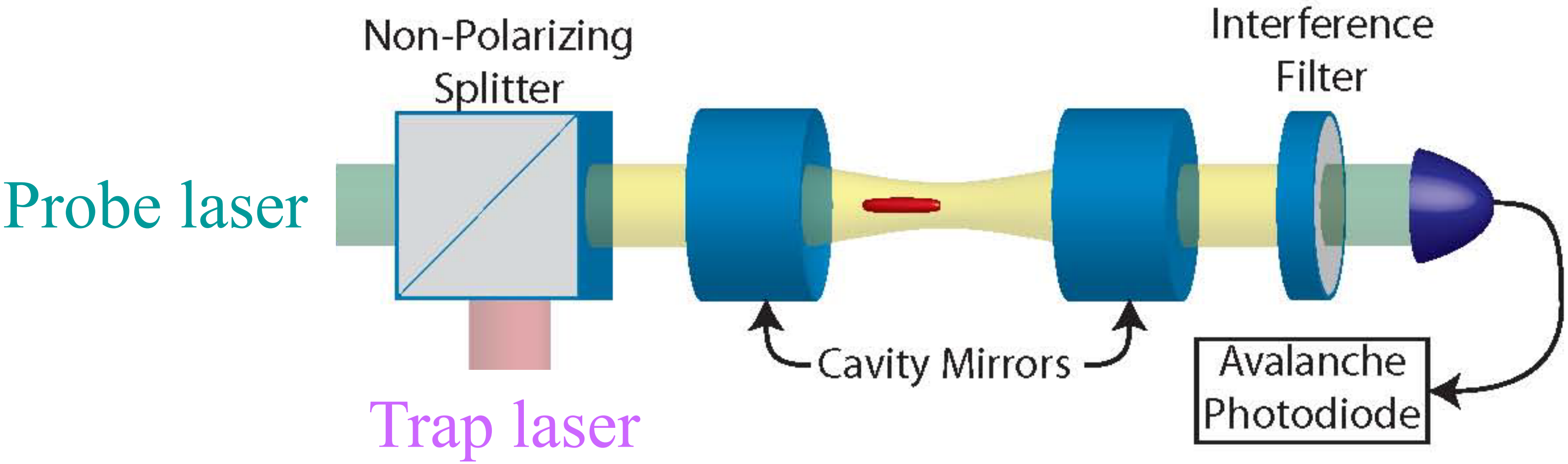
"one-axis twisting"

M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993);

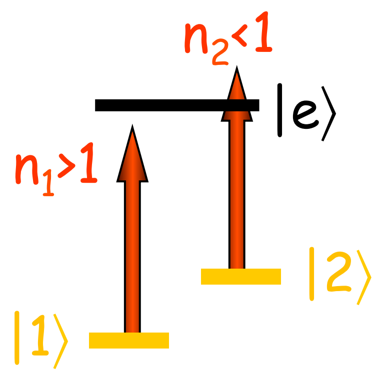
Creation of NOON states by spin squeezing around the Bloch sphere

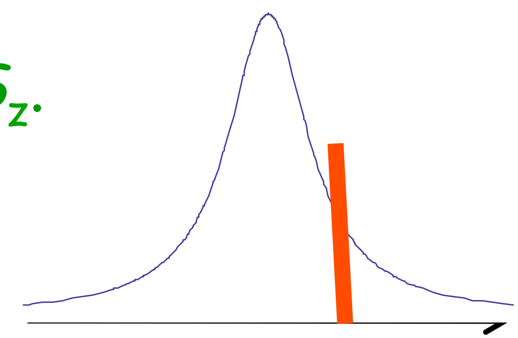


Setup for spin squeezing with light

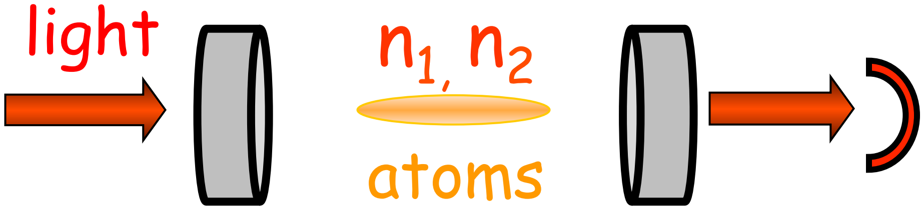


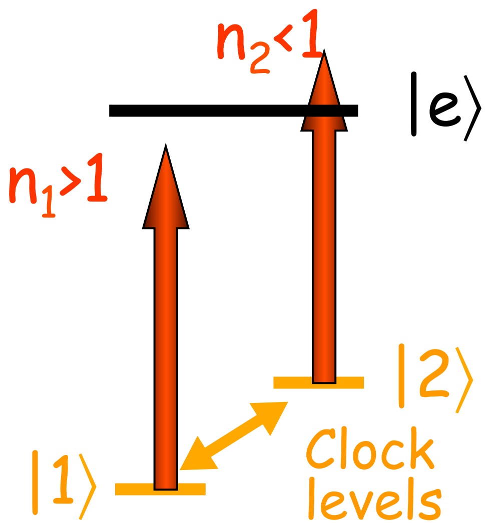
Intracavity probe power is proportional to S_z.

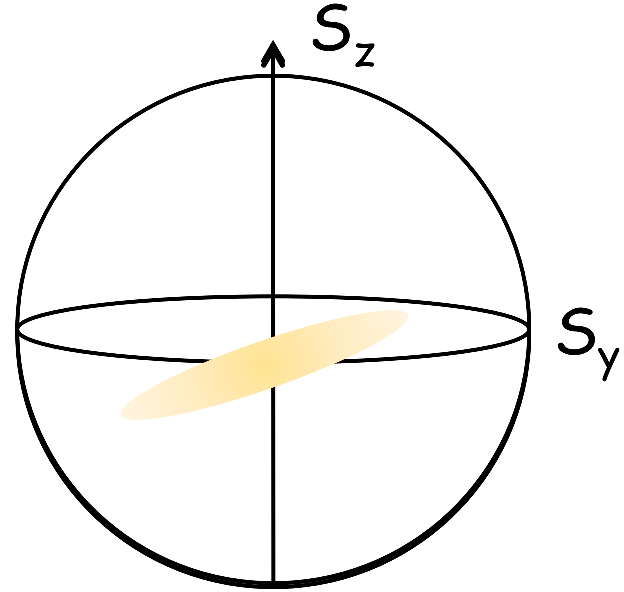


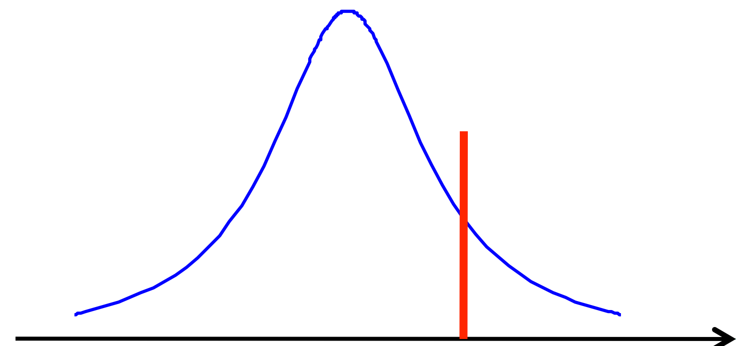


Spin squeezing by cavity feedback









S_z quantum noise is mapped onto light intensity that acts back on S_y variable: quantum correlation = spin squeezing

THE HAMILTONIAN

Intracavity photon number

Cavity shift by atoms or light shift by photons

Atomic energy

$$H = \hbar\omega_c c^\dagger c + \hbar\Omega c^\dagger c S_z + \hbar\omega_a S_z$$

Ω Differential light shift between atomic states per photon

$$(\mathrm{h}\omega_a + \mathrm{h}\Omega c^\dagger c)S_z$$

Or

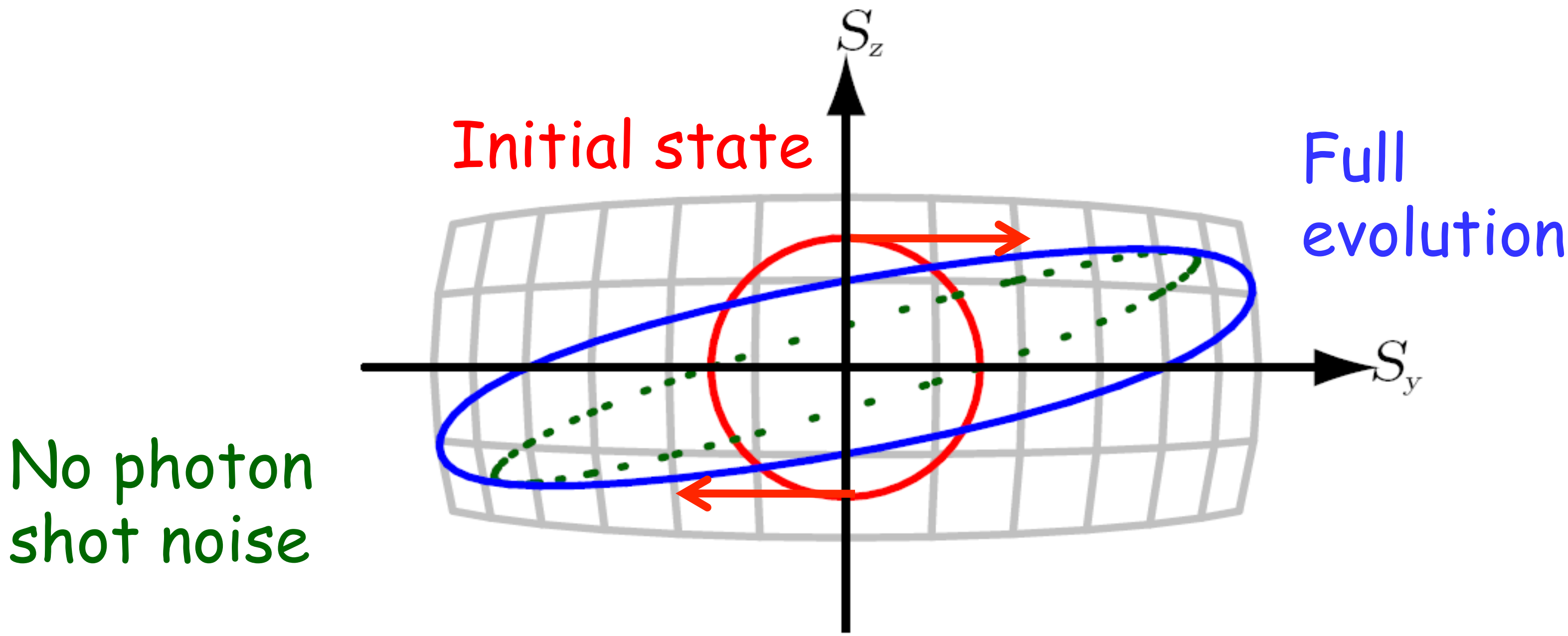
Differential cavity shift per atom (cavity shift when one atom changes state)

$$(\mathrm{h}\omega_c + \mathrm{h}\Omega S_z)c^\dagger c$$

Dynamic squeezing: theory

$$\dot{S}_+ = i\hbar^{-1}[H_0, S_+] = i\Omega c^\dagger c S_+$$

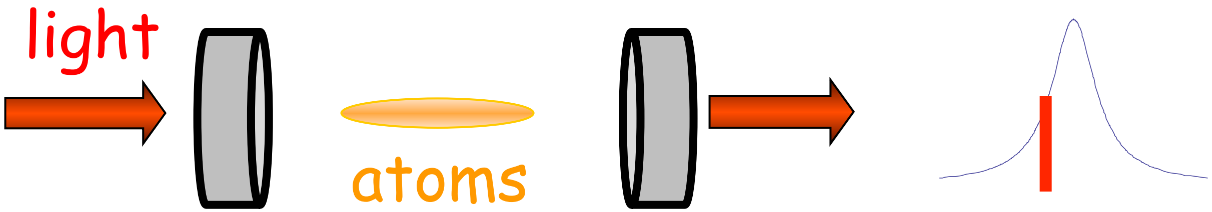
$$\dot{c} = \left(-\frac{\kappa}{2} - i\omega_c - i\Omega S_z\right) c + \sqrt{\kappa} b_{in}$$



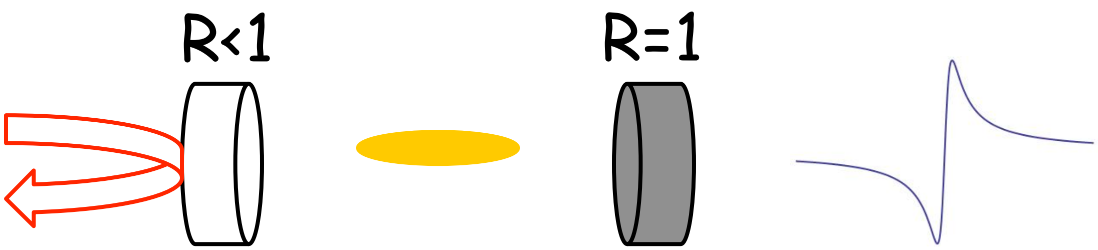
M. Schleier-Smith, I. Leroux, and V. Vuletic, PRA **81**, 021804(R) (2010).

Unitary spin squeezing: quantum eraser for light

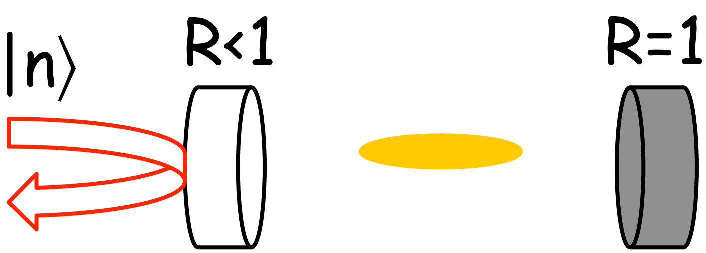
Atom-light entanglement leads to decoherence



Transmitted power depends on atomic state S_z



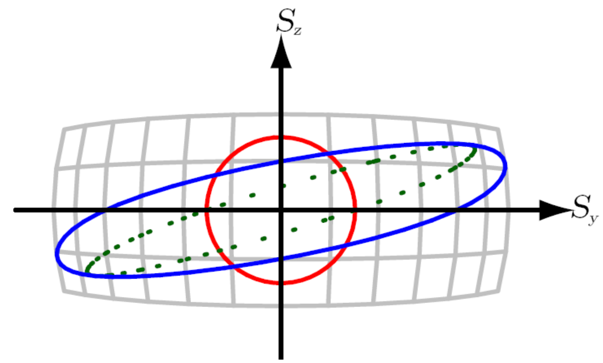
One-sided cavity: all power reflected, but S_z information in reflected phase

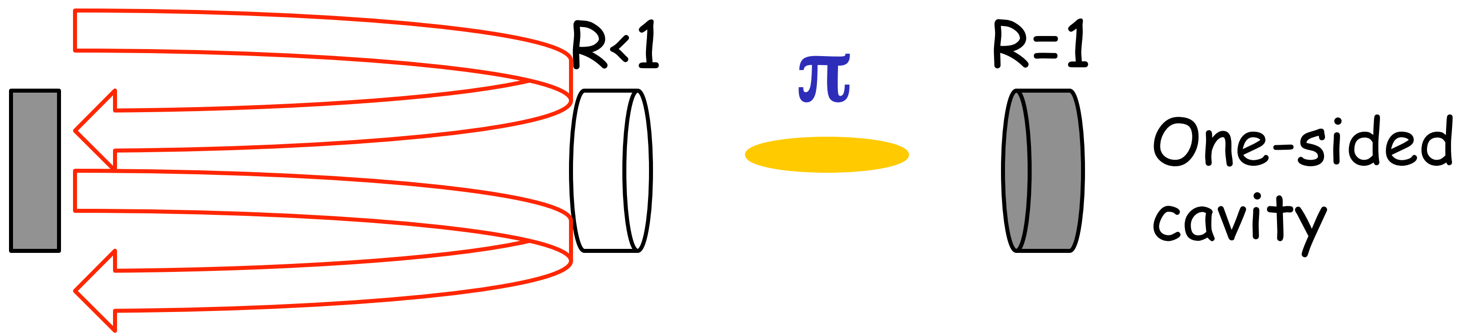


For photon number state input, there is no phase information in light

Spin echo erasure of photon shot noise

Photon shot noise in incident light broadens spin distribution when traced over incident light





Use spin echo to cancel photon shot noise while keeping squeezing term: interact twice with same pulse of light with atomic π pulse in between

I. D. Leroux, M. H. Schleier-Smith, and V. Vuletic, submitted;

Similar idea in free space:

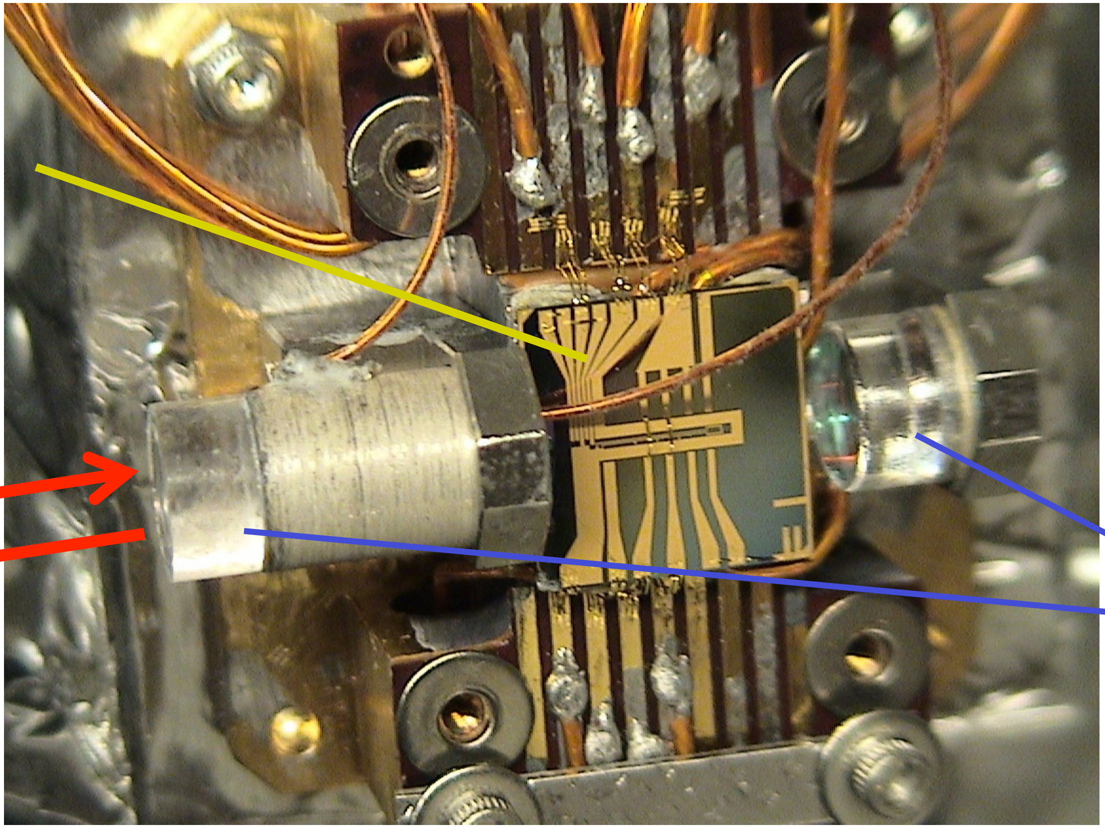
Trail, Jessen, and Deutsch, Phys. Rev. Lett. **105**, 193602 (2010).

Measurement of atom number in mesoscopic ensembles

Magnetic microchip for trapping ^{87}Rb atoms

Light in

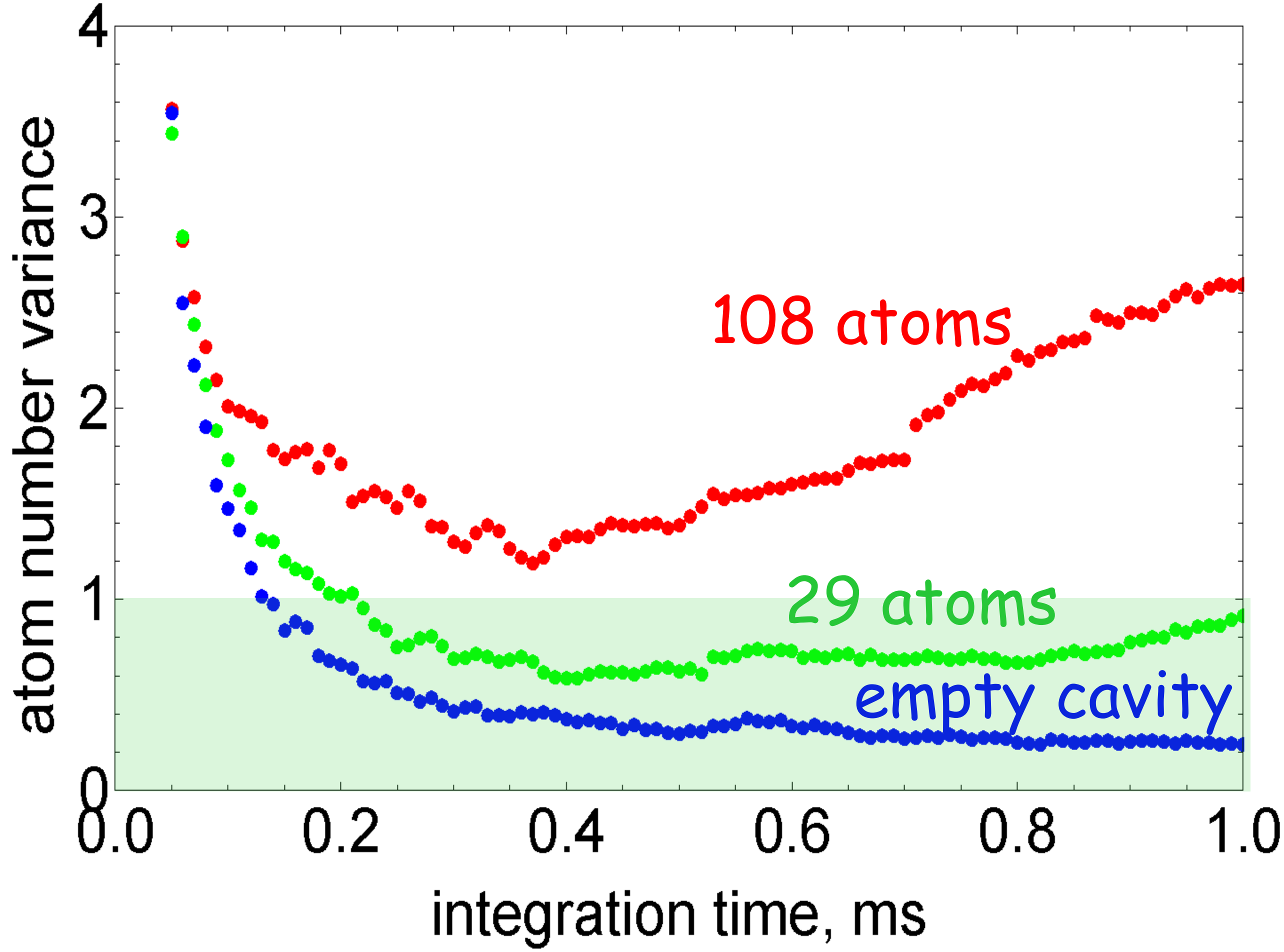
To detector



Optical resonator

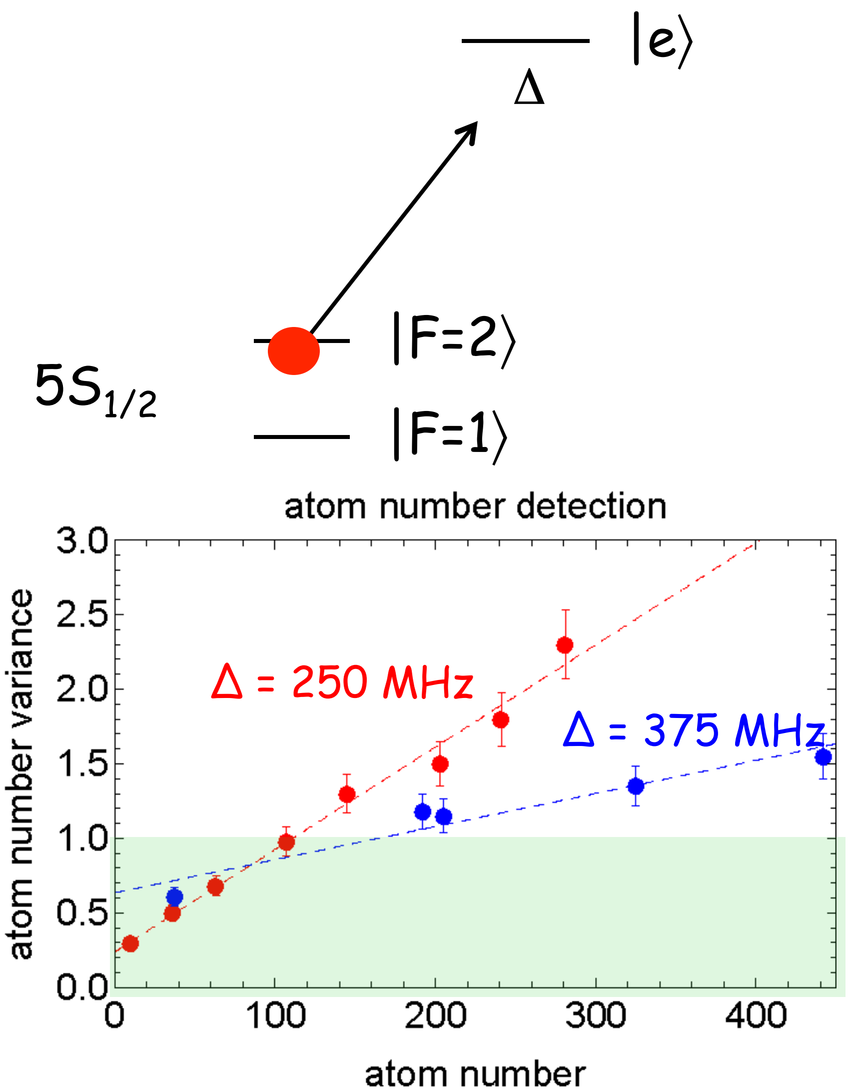
Atoms in optical trap 200 μm from the chip surface.
Temperature ~50 μK.

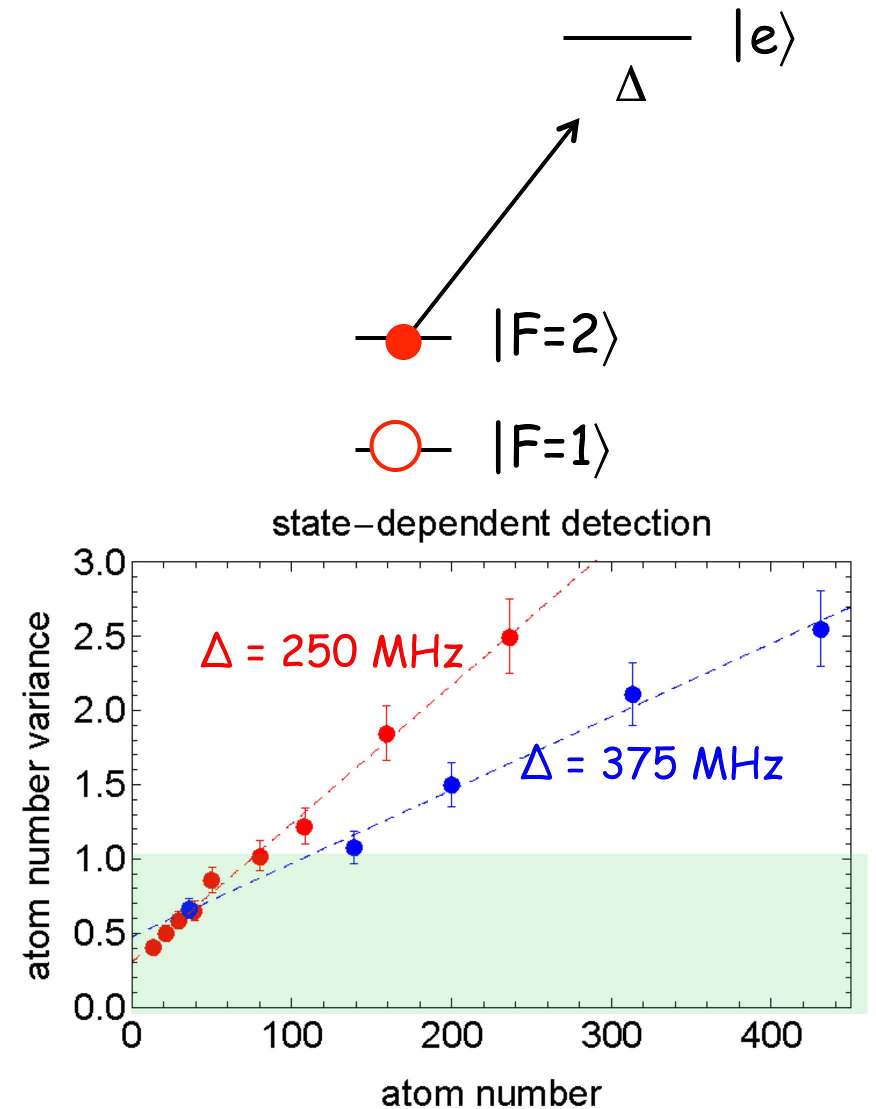
Atom number variance vs. integration time



Measurement of atom-induced cavity shift via Pound-Drever-Hall signal

State-independent and state-dependent counting of atoms vs. ensemble atom number



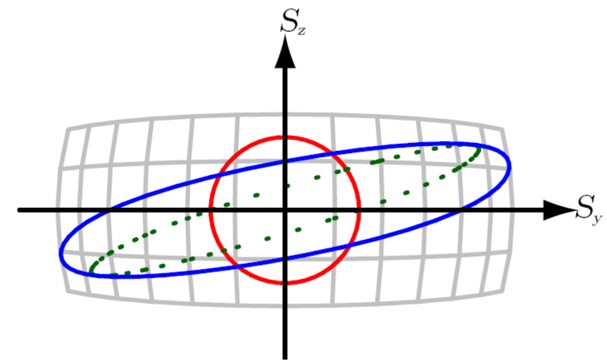


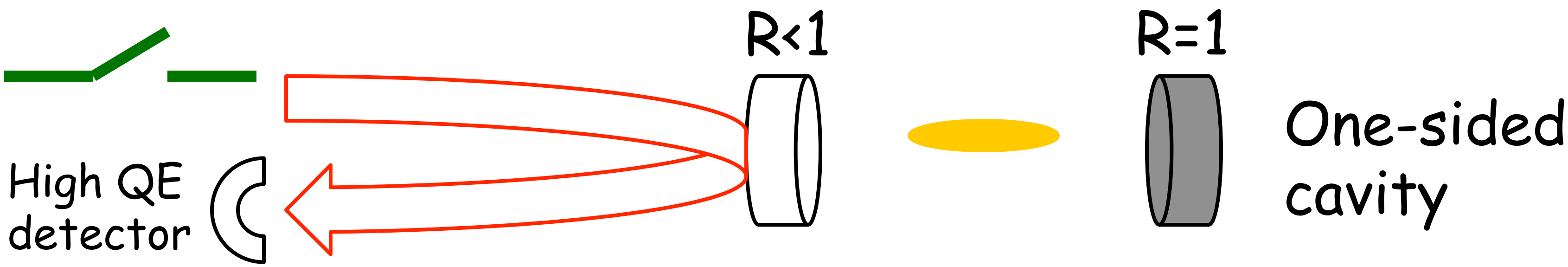
Summary

- We have demonstrated atom counting with single-atom resolution for mesoscopic ensembles up to ~ 100 atoms;
- Experimentally demonstrated spin squeezing in ensembles of distant atoms induced by light;
- Theoretically developed an experimentally viable setup for disentangling the light and the atoms, allowing for unitary spin squeezing;
- Unitary spin squeezing should allow the creation of NOON states in mesoscopic ensembles;
- Single-atom detection enables Heisenberg-limited interferometry.

Classical and quantum erasure of photon shot noise

Photon shot noise in incident light broadens spin distribution when traced over incident light





High QE detector: effective creation of photon Fock state by feedback or postselection